Algebra Preliminaries for Calculus – Part One

1. **Solve the equation:** $x^3 - x^2 - 6x = 0$

2. **Solve the equation:** $2x^2 - 6x + 3 = 0$

3. **Find the domain of the function** $f(x) = \sqrt{3x - 4}$. **Write your answer in interval notation.**

In items 4 and 5, solve the inequality and write the solution in interval notation.

4. $x^3 - 9x < 0$

5. $\dfrac{x^2 - 6x + 5}{(x - 3)^2} \leq 0$

In items 6 through 9, simplify the given expression.

6. $\dfrac{x^2 + 3x - 4}{x - 1}$

7. $\dfrac{\dfrac{1}{x + h} - \dfrac{1}{x}}{h}$

8. $\dfrac{f(x+h) - f(x)}{h}$, **given that** $f(x) = x^2 - x$

9. $\dfrac{3}{\sqrt{x+2} - \sqrt{x}}$ **(Rationalize the denominator.)**

10. **Find the slope of the line through the points with x-coordinates of 1 and 4, and y-coordinates given by** $y = f(x) = 4x^{-1/2}$.

11. **Find the equation of the line through point (3,1) with slope** $m = 2$. **Write your answer in slope-intercept form** $(y = mx + b)$.

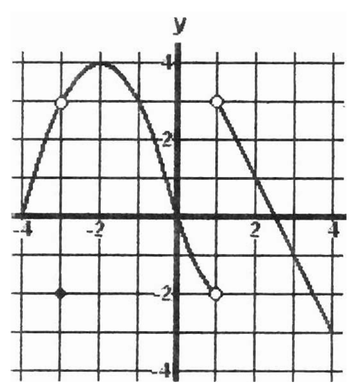
12. **Use the graph of function f to complete the following:**

$f(-3) =$ ______ $f(-1) =$ ______

$f(1) =$ ______ $f(3) =$ ______

f is increasing in the intervals: ____________________

f is decreasing in the intervals: ____________________



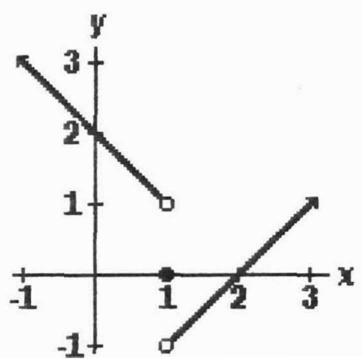
13. **Graph the function:** $f(x) = \begin{cases} 2-x & \text{if } x < 1 \\ 0 & \text{if } x = 1 \\ x-2 & \text{if } x > 1 \end{cases}$

14. **Graph the function:** $f(x) = \dfrac{|x+1|}{x+1}$

Answers

1. $x = 0, -2, 3$
2. $x = (3 \pm \sqrt{3})/2$
3. $[\,4/3\,,\,\infty\,)$
4. $(\,-\infty\,,\,-3\,) \cup (\,0,\,3\,)$
5. $[\,1\,,\,3\,) \cup (\,3\,,\,5\,]$
6. $x + 4$
7. $-\dfrac{1}{x(x+h)}$
8. $2x + h - 1$
9. $\dfrac{3\,(\sqrt{x+2} + \sqrt{x}\,)}{2}$
10. $-2/3$
11. $y = 2x - 5$
12. $f(-3) = -2$; $f(-1) = 3$; $f(1)$ is not defined; $f(3) = -1$; increasing in $[-4,-3)$ and $(-3,-2]$; decreasing in $[-2,1)$ and $(1,4]$

13.



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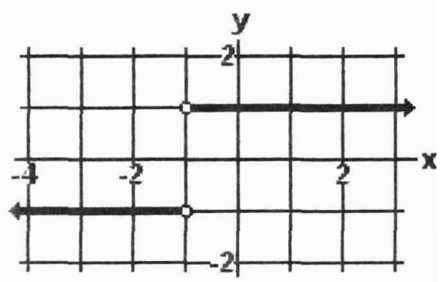


Algebra Preliminaries for Calculus – Part Two

1. **Complete the factoring of these expressions. Find and simplify the missing factor.**

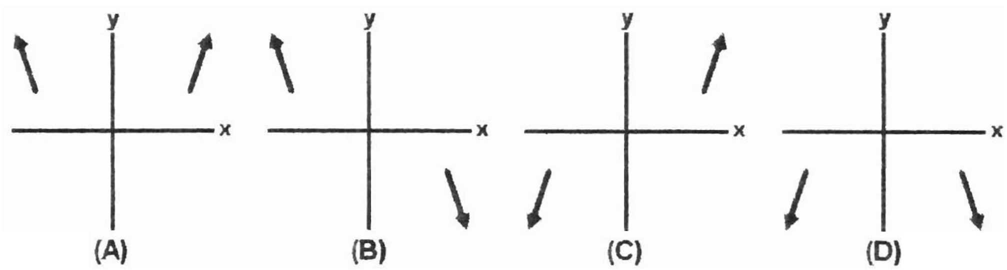
   a) $(2)(x^2 + 2)(2x)(1 - 2x^2)^2 + (x^2 + 2)^2(2)(1 - 2x^2)(-4x) = 4x\,(x^2 + 2)(1 - 2x^2)(\quad ? \quad)$

   b) $2x\,(x^2 - 1)^{1/2} + x^2(1/2)(x^2 - 1)^{-1/2}(2x) = x\,(x^2 - 1)^{-1/2}(\quad ? \quad)$

2. **Solve the equation:** $(4/3)x^{-2/3} - (2/3)x^{-1/3} = 0$

3. **Solve the equation:** $(x/6)(16 + x^2)^{-1/2} - 1/10 = 0$

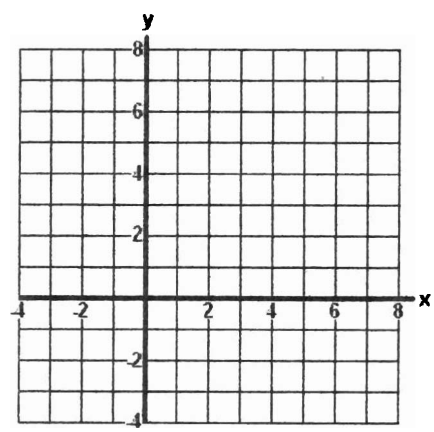
4. **Which of these graphs shows the end behavior of the function** $f(x) = x^2 + x^3 - x$ **?**



5. **Give functions f and g so that composite function** $f[g(x)] = \sqrt{x^2 + 3}$

   f(x) = ________; g(x) = __________

6. **Graph both** $f(x) = 2^x$ **and** $g(x) = \log_2 x$ **on this coordinate system.**



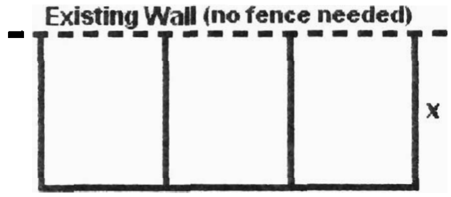
7. **Given function** $f(x) = e^x$, **find function g so that** $f[g(x)] = b^x$ **for** $b > 0$.

   g(x) = __________

8. **Given that** $\log_b 2 = 0.4479$ **and** $\log_b 3 = 0.7099$, **find** $\log_b 6$ **and** $\log_b 8$. **Give your answers correct to four decimal places.**

   $\log_b 6$ = _________; $\log_b 8$ = __________

Items 9 and 10 refer to this figure, which shows new fence (the solid lines) used to enclose a rectangular area and divide it into three smaller areas.



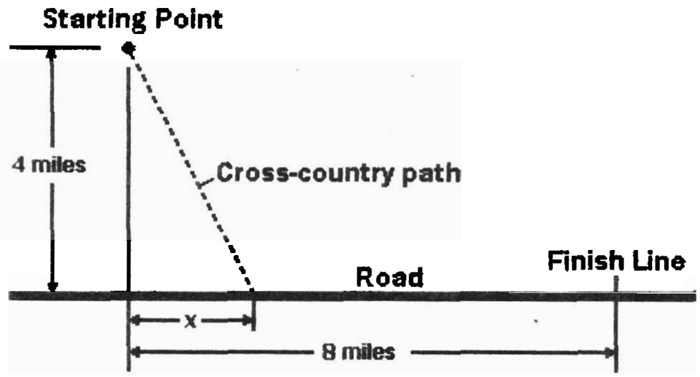
9. Given that the total length of new fence is 120 feet, express the total enclosed area as a function of the dimension "x" shown in the figure.

Area (in square feet) =

10. Given that the total enclosed area is 900 square feet, express the total length of new fence as a function of the dimension "x" shown in the figure.

Length (in feet) =

11. A runner averages six miles per hour on the cross-country path shown in the figure, then finishes the run on a road where he averages ten miles per hour. Express the total time of the run as a function of the dimension "x" shown in the figure.



Time (in hours) =

<u>Answers</u>

1a) $(-4x^2 - 3)$

1b) $(3x^2 - 2)$

2. $x = 8$

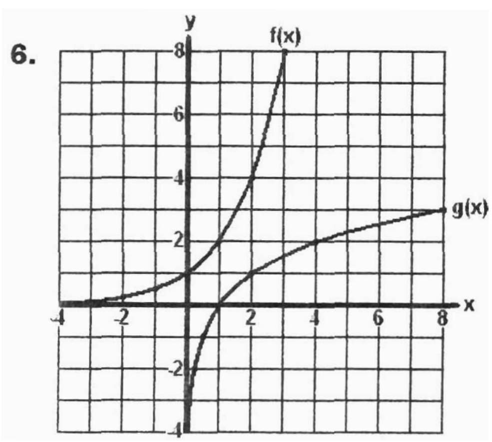
3. $x = 3$

4. C

5. $f(x) = \sqrt{x}$

$g(x) = x^2 + 3$

6.



7. $g(x) = x \ln b$, or

$g(x) = \ln b^x$

8. $\log_b 6 = 1.1578$;

$\log_b 8 = 1.3437$

9. Area $= x(120 - 4x)$

10. Length $= 4x + 900/x$

11. Time $= \dfrac{\sqrt{16 + x^2}}{6} + \dfrac{8 - x}{10}$